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Class Groups of Complex Quadratic Fields

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Abstract. We present 75 new examples of complex quadratic fields that have 5-rank of their class groups $\geqslant 3$. Only one of these fields has 5-rank of its class group > 3: The field $\mathbf{Q}(\sqrt{-258559351511807})$ has a class group isomorphic to

$$C(5) \times C(5) \times C(5) \times C(5) \times C(2) \times C(11828).$$

The fields were obtained by applying ideas of J. F. Mestre to the 5-isogeny $X_1(11) \to X_0(11)$.

1. Introduction. For any, multiplicatively written, finite abelian group A and any prime p, we define the p-rank of A, $d_p A = \dim_{\mathbf{F}_p} A/A^p$: the number of generators of the p-primary part of A. For any number field K, we denote by $\Delta(K)$ its absolute discriminant and by $\mathrm{Cl}(K)$ its ideal class group: a finite abelian group. The cyclic group of order n is denoted by $C(n)$. In the past decade some effort has been made to construct complex quadratic fields K with large $d_p\,\mathrm{Cl}(K)$ for *odd* prime p. Many examples of class groups with 3-rank = 3 and 3-rank = 4 have been found by Shanks and others [2], [3], [9] and, fairly recently, Solderitsch [10] gave examples of complex quadratic fields K with $d_5\,\mathrm{Cl}(K) = 3$, and one example with $d_7\,\mathrm{Cl}(K) = 3$. Also, in [4] Diaz y Diaz gave an example of a complex quadratic field K with $d_5\,\mathrm{Cl}(K) = 3$, that has a comparatively small discriminant. In this paper we present 74 complex quadratic fields K with 5-rank of their class groups equal to 3 and one example with 5-rank of its class group equal to 4. We obtained these examples by computing the class groups of 356 complex quadratic fields; the discriminants of these fields are parametrized by an 8th-degree polynomial $M(t) \in \mathbf{Z}[t]$.

2. The Polynomials $M(t)$. In this section we will explain the construction of polynomials, $M(t) \in \mathbf{Z}[t]$, that we use to parametrize a series of complex quadratic fields with class groups having p-rank $\geqslant 2$, for some prime p. The ideas involved are due to J. F. Mestre and are in [5]; here we only give the formulae to compute the polynomials $M(t)$.

Let p be a prime and F an elliptic curve defined over $\mathbf{Q}$ with a $\mathbf{Q}$-rational point P of order p on it. By E we denote the elliptic curve $F/\langle P\rangle$, which is, again, defined over $\mathbf{Q}$. We denote the isogeny $F \to E$ by φ. Let Q be a point on E with coordinates in some algebraic number field K. Then the coordinates of the points in $\varphi^{-1}(Q)$ generate an extension of K that is unramified over K and cyclic of degree p, provided that Q is submitted to certain conditions, cf. [5, Proposition II.1.3 and Proposition II.3.3]. To obtain quadratic fields with class groups having a p-rank $\geqslant 2$, one tries to find two distinct points Q_1 and Q_2 on E with coordinates in a quadratic number

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field K; under certain conditions, the points in the fibres $\varphi^{-1}(Q_1)$ and $\varphi^{-1}(Q_2)$ generate two *independent* unramified cyclic extensions of degree p of K, which, by class field theory, implies that $C(p) \times C(p)$ is a quotient of $\mathrm{Cl}(K)$, whence $d_p \mathrm{Cl}(K) \geq 2$.

Assume that the curve E is given by an equation:

$$Y^2 = X^3 - \frac{c_4}{48}X - \frac{c_6}{864}, \qquad c_4, c_6 \in \mathbf{Z}.$$

Let $Q_1 = (\xi_1, \eta)$ and $Q_2 = (\xi_2, \eta)$ be two distinct points on the curve E with ξ_1, $\xi_2 \in \mathbf{Q}$ and, as a consequence, η in some quadratic number field. We wish to compute the field $\mathbf{Q}(\eta)$, which will play the role of the field K from above. We have that

$$\eta^2 = \xi_1^3 - \frac{c_4}{48}\xi_1 - \frac{c_6}{864} = \xi_2^3 - \frac{c_4}{48}\xi_2 - \frac{c_6}{864},$$

so

(1)
$$\xi_1^2 + \xi_1\xi_2 + \xi_2^2 = \frac{c_4}{48}.$$

Let ζ denote a primitive sixth root of unity. Then ζ satisfies $\zeta^2 - \zeta + 1 = 0$. Put

$$\theta = \frac{12}{1+\zeta}(\xi_1 + \xi_2\zeta) \in \mathbf{Q}(\zeta).$$

Then $\mathrm{Norm}(\theta) = c_4$, and one easily computes

$$\theta^3 + \bar{\theta}^3 = 12^3\xi_1\xi_2(\xi_1 + \xi_2).$$

It follows that

$$\eta^2 = \xi_1^3 - \frac{c_4}{48}\xi_1 - \frac{c_6}{864} = -\xi_1\xi_2(\xi_1 + \xi_2) - \frac{c_6}{864} = -\frac{\bar{\theta}^3 + \theta^3}{12^3} - \frac{c_6}{864}.$$

So

(2)
$$(72\eta)^2 = -3\left(\theta^3 + \bar{\theta}^3 + 2c_6\right),$$

and the field $\mathbf{Q}(\eta)$ can be written as

$$\mathbf{Q}(\eta) = \mathbf{Q}\left(\sqrt{-3\,\mathrm{Trace}\left(\theta^3 + c_6\right)}\right).$$

From this representation of $\mathbf{Q}(\eta)$ it is plain that the numbers θ, $\zeta^2\theta$, $\zeta^{-2}\theta$, $\bar{\theta}$, $\zeta^2\bar{\theta}$, $\zeta^{-2}\bar{\theta}$ all give the same field $\mathbf{Q}(\eta)$.

Next we parametrize the conic (1), and we obtain a parametrization of the family of fields $\mathbf{Q}(\eta)$. To make sure that the equation (1) describes a nonempty curve over $\mathbf{Q}$, we will make the assumption that c_4 is a norm of an element of $\mathbf{Q}(\zeta)$. The numbers θ with $\mathrm{Norm}(\theta) = c_4$ can be parametrized, e.g., by

(3)
$$\theta(t) = (a + b\zeta)\left(\frac{2t+1}{t^2+t+1} + \frac{t^2-1}{t^2+t+1}\zeta\right), \qquad t \in \mathbf{Q} \cup \{\infty\},$$

where $a, b \in \mathbf{Z}$ and $a + b\zeta$ is a fixed number such that $\mathrm{Norm}(a + b\zeta) = c_4$, i.e., $a^2 + ab + b^2 = c_4$.

Substituting (3) in (2) eventually gives us that $\mathbf{Q}(\eta) = \mathbf{Q}(\sqrt{M(t)})$ with $M(t) \in \mathbf{Z}[t]$ of degree 8. The polynomial $M(t)$ can be computed as follows: Let

$$\begin{aligned} \alpha &= -a + b, & \mu_0 &= \nu_1 - 2c_6, \\ \beta &= -a - 2b, & \mu_1 &= -2\nu_3 - 6c_6, \\ \gamma &= 2a + b, & \mu_2 &= 5\nu_4 - 12c_6, \\ \nu_1 &= \alpha\beta\gamma, & \mu_3 &= -2\nu_2 - 14\nu_1 - 14c_6, \\ \nu_2 &= \alpha^3 + \beta^3 + \gamma^3, & \mu_4 &= 5\nu_3 - 12c_6, \\ \delta_3 &= \alpha^2\beta + \beta^2\gamma + \gamma^2\alpha, & \mu_5 &= -2\nu_4 - 6c_6, \\ \nu_4 &= \alpha^2\gamma + \beta^2\alpha + \gamma^2\beta, & \mu_6 &= \nu_1 - 2c_6. \end{aligned}$$

Then

$$M(t) = 3(t^2 + t + 1) \sum_{i=0}^{6} \mu_i t^i \in \mathbf{Z}[t].$$

Of course, $M(t)$ depends upon the choice of the number $a + b\zeta$. If $t \in \mathbf{Q} \cup \{\infty\}$, then the rational numbers (or ∞)

$$t,\ -1 - \frac{1}{t},\ -\frac{1}{t+1},\ \frac{-bt + a}{(a+b)t + b},\ \frac{at + (a+b)}{bt - a},\ -\frac{(a+b)t + b}{at + (a+b)}$$

all give, up to a square, the same value for $M(t)$, i.e. these numbers give the same field $\mathbf{Q}(\eta)$.

In order to insure that the class group of $\mathbf{Q}(\eta)$ has p-rank $\geqslant 2$, one submits the points Q_1 and Q_2 to certain conditions (cf. [5, Proposition II.2.2]). Numerical experience suggests that we should only bother about one of these:

(4) "the points Q_1 and Q_2 should not become singular modulo any prime of K."

This condition boils down to simple congruence conditions on t modulo primes that divide the conductor of E.

In the next section we use the formulae given above to obtain quadratic fields having class groups with p-rank $\geqslant 3$, for $p = 5$. At present, it seems unclear why such a large fraction of the computed class groups has a 5-rank greater than 2.

3. The Computations. We apply the formulae from Section 2 to the 5-isogeny $X_1(11) \to X_0(11)$. An equation for $X_0(11)$ can be found in [6, p. 82]:

$$Y^2 + Y = X^3 - X^2 - 10X - 20.$$

So, in the notation of [6, p. 36], we have that

$$(a_1, a_2, a_3, a_4, a_6) = (0, -1, 1, -10, -20),$$

whence (in the notation of [6, p. 36]) $c_4 = 496$ and $c_6 = 20008$. Now c_4 is a norm from $\mathbf{Q}(\zeta)$, e.g. Norm $(20 + 4\zeta) = c_4$, and we take

$$a = 20 \quad \text{and} \quad b = 4.$$

A straightforward computation, using the formulae given in the previous section, gives us that, up to a square,

$$M(t) = -(t^2 + t + 1) \cdot (47t^6 + 21t^5 + 598t^4 + 1561t^3 + 1198t^2 + 261t + 47),$$

which is, up to a linear transformation, Mestre's polynomial $m(t)$ in his Proposition II.2.2 in [5].

The conductor of $X_0(11)$ equals 11 and condition (4) here boils down to

$$t \not\equiv 2, -4, 4 \pmod{11}. \tag{5}$$

(i.e. if $t = p/q \in \mathbf{Q} \cup \{\infty\}$ with $p, q \in \mathbf{Z}$, $\gcd(p, q) = 1$, then $p \not\equiv 2q, -4q, 4q \pmod{11}$).

We computed the class groups of the fields $\mathbf{Q}(\sqrt{M(t)})$ for $t \in \mathbf{P}_1(\mathbf{Q})$ with:

(i) t satisfies (5).

(ii) $t = p/q$; $p, q \in \mathbf{Z}$ with $|p|, |q|, |p+q| \leqslant 40$.

This comprised 356 nonisomorphic complex quadratic fields, and we obtained the following distribution of the 5-primary parts of their class groups:

5-primary part	freq.	5-primary part	freq.
$C(5)$	1	$C(5) \times C(5) \times C(5)$	55
$C(5) \times C(5)$	210	$C(5) \times C(5) \times C(25)$	14
$C(5) \times C(25)$	51	$C(5) \times C(5) \times C(125)$	3
$C(5) \times C(125)$	17	$C(5) \times C(5) \times C(625)$	2
$C(5) \times C(625)$	1	$C(5) \times C(5) \times C(5) \times C(5)$	1
$C(5) \times C(3125)$	1		
total	218	total	75

For the following 75 values of t, the field $\mathbf{Q}(\sqrt{M(t)})$ has 5-rank of its class group $\geqslant 3$:

$$\frac{1}{4}, \frac{1}{5}, \frac{7}{2}, \frac{2}{7}, \frac{7}{4}, \frac{7}{5}, \frac{1}{11}, \frac{11}{3}, \frac{3}{11}, \frac{2}{13}, \frac{5}{11}, \frac{7}{33}, \frac{11}{6}, \frac{2}{15}, \frac{5}{12},$$
$$\frac{11}{7}, \frac{16}{3}, \frac{11}{26}, \frac{17}{2}, \frac{14}{5}, \frac{7}{11}, \frac{10}{9}, \frac{5}{14}, \frac{6}{13}, \frac{15}{7}, \frac{1}{21}, \frac{2}{21}, \frac{23}{1}, \frac{19}{7}, \frac{12}{13},$$
$$\frac{17}{9}, \frac{7}{18}, \frac{20}{7}, \frac{11}{15}, \frac{16}{11}, \frac{11}{16}, \frac{20}{17}, \frac{17}{11}, \frac{20}{9}, \frac{27}{2}, \frac{1}{27}, \frac{18}{11}, \frac{9}{19}, \frac{16}{13}, \frac{25}{6},$$
$$\frac{3}{26}, \frac{24}{7}, \frac{5}{24}, \frac{19}{11}, \frac{11}{18}, \frac{8}{21}, \frac{9}{20}, \frac{1}{29}, \frac{28}{5}, \frac{21}{11}, \frac{1}{31}, \frac{20}{13}, \frac{32}{1}, \frac{19}{14}, \frac{17}{16},$$
$$\frac{2}{31}, \frac{33}{1}, \frac{33}{2}, \frac{23}{12}, \frac{11}{23}, \frac{29}{8}, \frac{27}{10}, \frac{12}{23}, \frac{35}{3}, \frac{34}{5}, \frac{2}{35}, \frac{1}{38}, \frac{14}{25}, \frac{39}{1}, \frac{11}{29}.$$

(The values of t are listed according to the size of the discriminants of the associated fields. Of course, every field occurs for at least six different values of t; we picked $t = p/q \geqslant 0$ with $|p+q|$ minimal.) We list all 75 fields with their class groups in the table and single out 16 fields for special mention.

The following is a list of all complex quadratic fields K, with $d_5 \mathrm{Cl}(K) = 3$ and $|\Delta(K)| < 10^{10}$, known to us:

	Δ(K)	factorization	t	h	5-part	rest	$L(1, \chi)$
1.	18397407	3.7.876067	1/4	2000	5 × 5 × 5	2 × 8	1.465
2.	77778287	31.103.24359	1/5	6000	5 × 5 × 5	2 × 24	2.137
3.	205996583	13.73.131.1657		10000	5 × 5 × 25	2 × 2 × 4	2.189
4.	1156599359	47.67.311.1181	7/2	34000	5 × 5 × 5	2 × 2 × 68	3.141
5.	2048074559	67.3323.9199	2/7	52500	5 × 5 × 5	2 × 210	3.644
6.	7558314879	3.31.883.92041	7/4	60000	5 × 5 × 25	2 × 4 × 12	2.168

(Here and in the next table we denote by $n_1 \times n_2 \times \cdots \times n_T$ the abelian group $C(n_1) \times C(n_2) \times \cdots \times C(n_T)$.) Diaz y Diaz was the first to compute the class group of the fields 1, 2, and 3. The field 3 was found by him by an entirely different method [4].

	$-\Delta(K)$	factorization	t	h	5-part	rest	$L(1,\chi)$
7.	47	prime	0	5	5	1	2.291
8.	11199	3.3733	1	100	5×5	4	2.969
9.	258559351511807	1171.1439.153441403	14/25	1478500	$5\times 5\times 5\times 5$	2×11828	2.889
10.	222637549223	prime	7/33	434625	$5\times 5\times 5$	3477	2.894
11.	3513582927119	487.7214749337	2/21	2178000	$5\times 5\times 5$	3×5808	3.650
12.	37262495315279	13.61.46989275303	21/11	7749000	$5\times 5\times 5$	6×10332	3.988
13.	10368869999	97.106895567	3/8	118750	5×625	38	3.664
14.	1449192975839	7.61.163.20821439	19/3	1000000	5×3125	$2\times 2\times 16$	2.610
15.	4574009420324	2^2.97.2297.5132209	–	1088000	$5\times 5\times 5$	$2\times 4\times 1088$	1.598
16.	51887726858696	2^3.6485965857337	–	4492500	$5\times 5\times 5$	2×215252	1.959

The field 7: $\mathbf{Q}(\sqrt{-47})$ occurs for $t = 0$ (and $t = -1, \infty, 5, -\frac{6}{5}, \frac{1}{6}$); in the range of our computations it is the only value of t, satisfying the condition (5), for which the corresponding field has a class group with 5-rank < 2. The field 8: $\mathbf{Q}(\sqrt{-11199})$ is the smallest field K (small with respect to $|\Delta(K)|$) that has a class group whose 5-rank equals 2, cf. [1]. The next entry in our table, field 9, is the only example we found of a complex quadratic number field K with $d_5\,\mathrm{Cl}(K) = 4$. At present it is the only known example of a complex quadratic field possessing this property. We give four independent ideal classes of order 5 of this field K by giving the associated reduced binary quadratic forms of discriminant $\Delta(K)$. Recall that a reduced binary quadratic form $aX^2 + bXY + cY^2$ of discriminant $\Delta = b^2 - 4ac$ corresponds to the ideal class

$$\left\{\left(\mathbf{Z} + \frac{b+\sqrt{\Delta}}{2a}\mathbf{Z}\right)\cdot\alpha : \alpha \in K^{\times}\right\}$$

of $K = \mathbf{Q}(\sqrt{\Delta})$.

The four ideal classes correspond to:

$$\begin{aligned}
179988X^2 &+ 55577XY + 359138443Y^2,\\
7536956X^2 &+ 1954041XY + 8703037Y^2,\\
535437X^2 &+ 408245XY + 120801334Y^2,\\
4413782X^2 &+ 1926753XY + 14855272Y^2.
\end{aligned}$$

It is not difficult to check that these forms are actually of order 5 and independent, e.g. by using the formulae for composition of quadratic forms as given by Shanks in [8].

Example 10 is, apart from example 7, the only field with prime discriminant that we encountered in our search; it occurred for $t = 7/33$ (note that $7^2 + 7.33 + 33^2 = 37^2$).

The fields 11 and 12 are listed since they are "irregular" for both 3 and 5: the 5-rank of their class groups equals 3, while the 3-rank equals 2. The fields 13 and 14 have class groups with unusual 5-primary parts; these groups are isomorphic to $C(5) \times C(5^4)$ and $C(5) \times C(5^5)$, respectively. We encountered these types of class groups only once.

Finally two of the fields that Solderitsch found [10] are listed. These two fields have discriminants in the range of our computations; the absolute values of the discriminants of the other fields he found are much larger.

It is possible to do computations like these using other elliptic curves. However, if one uses elliptic curves that are defined over **Q**, one cannot apply this method for $p \geqslant 11$, since rational p-torsion points on elliptic curves do not exist if $p \geqslant 11$. We did some computations for $p = 7$, but did not succeed in finding new 7-rank = 3 examples.

The computations of the class groups have been done using Shanks's algorithm as described in [8]. A feature of this algorithm is that it is theoretically possible that one does not compute the full class group, but that one only finds a subgroup of the class group; it is extremely unlikely that this occurred in our computations, but, strictly speaking, all the values of the 5-ranks we found are, in fact, lower bounds.

-Δ(K)		t	h			L(1,χ)
18397407	3.7.876067	1/4	2000	5×5×5	2×8	1.465
77778287	31.103.24359	1/5	6000	5×5×5	2×24	2.137
1156599359	47.67.311.1181	7/2	34000	5×5×5	2×2×68	3.141
2048074559	67.3323.9199	2/7	52500	5×5×25	2×42	3.644
7558314879	3.31.883.92041	7/4	60000	5×5×25	2×4×12	2.168
16704202367	47.109.3260629	7/5	125000	5×5×625	2×4	3.038
19283393759	7.19.683.21281	1/11	173000	5×5×5	2×2×346	3.914
39246913919	163.240778613	11/3	219250	5×5×5	1754	3.477
69971761919	53.163.8099521	3/11	345000	5×5×25	2×276	4.097
116734226447	199.586604153	2/13	305000	5×5×25	488	2.804
208703173647	3.67.1038324247	5/11	211000	5×5×5	2×844	1.451
222637549223	prime	7/33	434625	5×5×5	3477	2.894
240820329839	223.2897.372769	11/6	579500	5×5×5	2×2318	3.710
315633202367	7.37.1218661013	2/15	579500	5×5×5	2×2318	3.240
338605831007	229.1478628083	5/12	552500	5×5×25	884	2.983
407654485199	13.19.199.1021.8123	11/7	848000	5×5×5	2×2×2×848	4.173
440024496719	313.1405829063	16/3	756000	5×5×5	6048	3.580
472440264519	3.199.257.311.9901	11/26	634000	5×5×5	2×2×2×634	2.898
477720858639	3.109.863.1692839	17/2	700000	5×5×125	2×2×56	3.182
488591920767	3.97.269.6241673	14/5	357000	5×5×5	2×2×714	1.605
526789501199	13.19.2132751017	7/11	991500	5×5×5	2×3966	4.291
719058505007	271.5527.480071	10/9	801000	5×5×5	2×3204	2.968
819641901567	3.53.97.401.132529	5/14	570000	5×5×25	2×2×2×114	1.978
825270838559	199.283.1123.13049	6/13	1207000	5×5×5	2×2×2414	4.174
1764613514207	379.3463.1344491	15/7	1004000	5×5×5	2×4016	2.374
2474580780719	463.1109.4819357	1/21	1884000	5×5×5	2×7536	3.762
3513582927119	487.7214749337	2/21	2178000	5×5×5	3×5808	3.650
4025744542799	7.79.401.18154183	23/1	1779000	5×5×5	2×2×3558	2.785
6078086981679	3.181.269.41611637	19/7	1751000	5×5×5	2×2×3502	2.231
6822526267487	7.67.14546964323	12/13	2362500	5×5×125	2×378	2.842

-Δ(K)		t	h			L(1,χ)
7111644846239	523.13597791293	17/9	3797500	5x5x25	6076	4.474
7391579442047	499.14812784453	7/18	2030500	5×5×5	16244	2.346
8065721968127	19.31.13693925243	20/7	2720000	5×5×25	2×2176	3.009
9795957818927	7.73.1873.10235009	11/15	2146000	5×5×5	2×2×4292	2.154
10799568953999	7.79.4093.4771331	16/11	4326000	5×5×5	2×2×8652	4.136
13375918976399	7.79.103.3613.64997	11/16	3392000	5×5×5	2×2×2×3392	2.914
13598357713967	13.43.24326221313	10/17	3069000	5×5×5	2×12276	2.615
14077525107999	3.53.199.444914039	17/11	3237000	5×5×5	2×2×6474	2.710
15826902503327	661.7789.3074063	20/9	2877500	5×5×25	2×2302	2.272
16009647635519	419.787.6917.7019	27/2	5085000	5×5×25	2×2×2034	3.993
17124593400479	757.8363.2704969	1/27	4022000	5×5×5	4×8044	3.053
18178141409279	643.883.32016791	18/11	5274000	5×5×5	2×21096	3.886
18355577207519	613.29943845363	9/19	6477750	5×5×5	51822	4.750
20434497658959	3.53.211.609094091	16/13	3525000	5×5×125	2×2×282	2.450
22226379018527	53.811.14851.34819	25/6	3834000	5×5×5	2×2×7668	2.555
22526019100319	7.109.199.419.354073	3/26	4190000	5×5×25	2×2×2×838	2.773
23031374411279	13.61.46817.620359	24/7	6926000	5×5×5	2×2×13852	4.534
23234046745007	7.103.115663.278609	5/24	4260000	5×5×25	2×4×852	2.776
23271228811967	691.33677610437	19/11	2965250	5×5×5	23722	1.931
24008715204479	643.37338592853	11/18	5808000	5×5×5	46464	3.724
24213534365039	53.103.673.6590677	8/21	6842000	5×5×5	2×2×13684	4.368
24307482796127	499.661.73694993	9/20	5068500	5×5×5	2×20274	3.230
29784718976207	13.67.160009.213713	1/29	5702000	5×5×5	2×2×11404	3.282
36573526186847	13.73.163.199.499.2381	28/5	5264000	5×5×5	2×2×2×2×2632	2.735
37262495315279	13.61.46989275303	21/11	7749000	5×5×5	6×10332	3.988
49985970332079	3.331.50338338703	1/31	6374000	5×5×5	2×25496	2.832
52508111150207	829.63339096683	20/13	4877500	5×5×25	7804	2.115
54805390012079	7.151.7517.6897691	32/1	7497000	5×5×5	2×2×14994	3.181
55329101911439	103.823.4723.138197	19/14	9997000	5×5×5	2×2×19994	4.222
60410353317359	19.43.53629.1378763	17/16	10724000	5×5×5	2×2×21448	4.335
63123375138239	13.79.103.269.2218351	2/31	9944000	5×5×5	2×2×2×9944	3.932
69948783320639	1123.62287429493	33/1	9438250	5×5×5	75506	3.545
76087582641167	19.61.1699.38639987	33/2	8228000	5×5×5	2×4×8228	2.963
76178156852447	13.73.103.9227.84463	23/12	8884000	5×5×5	2×2×2×8884	3.198
86754370349199	3.7.43.269.357150557	11/23	6098000	5×5×5	2×2×2×6098	2.057
93633351110319	3.379.82351232287	29/8	7569000	5×5×5	2×30276	2.457
102440524590047	7.157.93212488253	27/10	8060500	5×5×5	2×32242	2.502
109165179721247	13.53.73.883.2457997	12/23	8250000	5×5×625	2×2×2×66	2.481
133514240116127	13.103.18329.5440117	35/3	11046000	5×5×5	2×2×22092	3.003
143095169224847	7.53.193.1998452149	34/5	7972000	5×5×5	2×4×7972	2.094

$-\Delta(K)$		t	h			$L(1,\chi)$
157897435920447	3.47.433.2586235499	2/35	5090000	5×5×25	2×2×2036	1.273
244367110736159	1483.164778901373	1/38	19072750	5×5×5	152582	3.833
258559351511807	1171.1439.153441403	14/25	14785000	5×5×5×5	2×11828	2.889
263424263462927	7.47.223.10993.326617	39/1	13962000	5×5×5	2×2×2×13962	2.703
317159735746287	3.7.61.199.1244158873	11/29	7008000	5×5×5	2×2×2×7008	1.236

Two computer programs were used: one computes class groups of complex quadratic fields K given their discriminants $\Delta(K) > -2.5_{10}14$; the other is a double length version of this program [7]. All computations have been done on the CDC-computer system of SARA in Amsterdam.

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